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| March 16, 2013 | Matt Landreman |

# Normalizations for the multi-species pedestal code

The drift-kinetic equation for each species may be written



where ,

,

, and . Expanding the right-hand side of ,



Just as in the local code, we normalize using a position-independent temperature , density , potential , magnetic field , and length . We take  and  to be the same for all species. We thereby obtain the quantities



 (where )





and



so



for any function . Also, for each species,



and

.

We define a species-independent reference mass  and reference thermal speed

.

Each species then has a normalized mass



and thermal speed

.

Two combinations of the normalization quantities will appear in the dimensionless equations:



(which resembles ) and



.

The normalized distribution function of each species  is defined by

.

Notice the normalization is the same for all species.

To normalize the kinetic equation, we first multiply through by



where the temperature dependence is included for historical reasons, for consistency with the single-species code. We then obtain



where



is the normalized source, and



is the right-hand side.

We next change the independent variables in the drift-kinetic operator from  to , to re-write in the form

.

We begin with the streaming term:







.

Moving on to the drift terms, we begin with

.

which gives

.

This result differs by a factor  from the single-species code.

Next, we evaluate the contribution to :

.

We can re-use two results from the single-species calculation:



and

.

Then becomes

.

This result is equal to (59) in the single-species technical manual, but with an extra factor of  in each of the magnetic drift terms. Combining with , we obtain

.

This equation is the generalization of (60) in the single-species technical manual, with the new factors colored red.

To obtain , we first write

.

Differentiating,

.

Applying ,

.

To obtain , we first write



Differentiating,



Solving for ,



Plugging in our previous results,



Two pairs of terms cancel, leaving

.

## Collision operator

The linearized collision operator may be written

,

where the Lorentz part of the collision term is



with





.

The energy scattering contribution is



where

.

The field term is



where the potentials are defined by



and

.

We write the field term as



where







The Poisson equations that define the potentials are



.

Let us define





so the defining equations for the potentials become



.

It is convenient to specify the collisionality at the reference parameters:



where



The kinetic equation may then be written

.

Next, it is convenient to note



Then we may write the kinetic equation as



where

.

Expanding  as before,



where



The energy scattering component is



The diagonal term is



In the cross-species case, this term is no longer identical to the  term in energy scattering (as it is in the same-species case).

The  term in the collision operator is



Although in principle we would also be free to write



(i.e. replacing  in two places), the resulting expression is less convenient because we compute  on the  grid, and so it is easier to differentiate with respect to .

The  collision term is



## Right-hand side

Applying



to and using the identity

,

we find the right-hand side of the linear system is

.

## Outputs

### Normalized scale lengths







### Normalized electric field



### Density perturbation



### Parallel flow



where

.

### Mach number

.

### Pressure perturbation



### Average density perturbation



### Average parallel flow



where

.

### Parallel flow coefficient







### Average pressure perturbation



### Particle flux

Define

.

Recall



Manipulation gives

.

We may apply the identity



to obtain

.

The dimensionless “particleFlux” quantity computed in the code is



so

.

### Momentum flux

Define a momentum flux  by

.

Manipulation gives

.

We may apply the identity

.

Thus, the dimensionless quantity computed in the code is

.

where

.

### Heat flux

Define a heat flux  by

.

Manipulation gives

.

We may again apply . The dimensionless quantity computed in the code is



so

.

### Perturbed potential

Assuming a pure plasma in which only the ions are simulated in PERFECT, the quasineutrality equation is



where  is the unperturbed density. Further assuming , then



so

.